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Dynamic Response Analysis of Stochastic Frame Structures Under Nonstationary Random Excitation

W. Gao,* J. J. Chen,† J. Ma,‡ and Z. T. Liang‡
Xidian University, 710071 Shaanxi, People's Republic of China

A new method (random factor method) for the dynamic response analysis of linear stochastic frame structures under nonstationary random excitation is presented. From the expressions of structural random response of the frequency domain, the computational expressions of the mean value, variance, and variation coefficient of the mean square value of the structural nonstationary random displacement and stress response are developed by means of the random variable's functional moment method and the algebra synthesis method, in which the randomness of the structural physical parameters is considered. The influences of the randomness of the structural physical parameters on the randomness of the mean square value of the structural displacement and stress response are inspected via an engineering example.

Nomenclature

$[B]$ = element's strain matrix
$C_{E\rho}$ = correlation coefficient of variables elastic modulus and mass density
$[C]$ = damping matrix
$[D]$ = structural elastic matrix
$\mathrm{diag}(\cdot)$ = diagonal matrix with diagonal elements given in parentheses
$[E]$ = elastic modulus
$g(t)$ = time modulation function
$[H(\omega)]$ = frequency response function matrix
$[H^*(\omega)]$ = conjugate matrix of $[H(\omega)]$
$[h(t)]$ = impulse response function matrix
$[K]$, $[M]$ = global stiffness and mass matrices, respectively
$[K^{(e)}]$, $[M^{(e)}]$ = eth element's stiffness and mass matrices, respectively
$[\hat{K}^{(e)}]$ = eth element's stiffness matrix in global coordinate
n = number of the natural frequencies
n_e = number of the structural elements
$\{\boldsymbol{P(t)}\}$ = stationary random excitation vector
$[R_p(t_1 - t_2)]$ = correlation function matrix of the $\{\boldsymbol{P(t)}\}$
$[R_u(t_1, t_2)]$ = correlation function matrix of structural displacement response
$[R_\sigma^{(e)}(t_1, t_2)]$ = correlation function matrix of the eth element's stress response
$[S_P(\omega)]$ = equivalent one-side power spectral density matrix of $\{\boldsymbol{P(t)}\}$
$[S_u(\omega)]$ = power spectral density matrix of displacement response
$[S_\sigma^{(e)}(\omega)]$ = power spectral density matrix of the stress response of the eth element
$\{\boldsymbol{u(t)}\}$, $\{\dot{\boldsymbol{u}}(t)\}$, $\{\ddot{\boldsymbol{u}}(t)\}$ = displacement, velocity, and acceleration vectors, respectively
$\{u(t)^{(e)}\}$ = displacement response of the nodal point of the eth element
μ = mean value of random variable
ν = variation coefficient of random variable
ζ_j = jth-order mode damping of structure
ρ = mass density
σ = mean variance of random variable
$[\phi]$ = normal modal matrix
$[\psi_u^2]$ = mean square value matrix of structural displacement response
ψ_{uk}^2 = mean square value of the kth degree of freedom of dynamic displacement response
$[\psi_{\sigma^{(e)}}^2]$ = mean square value matrix of the eth element stress response
ω = natural frequency

I. Introduction

THE analysis of structures with deterministic characteristic to random excitations has been reported extensively in the literature. Nigam and Narayanan[1] considered different types of loading in this group of problems. However, structural analysis with random material properties has not been developed to the same extent. As a matter of fact, the randomness of structure exists objectively and must be considered. For example, for numerous or batch producing structures, the values of their material physical parameters have randomness. In many realistic engineering examples, applied loads are random process forces; some of them can be treated as stationary random process excitations, but earthquakes, blast shocks, and hurricanes must be considered nonstationary random process excitations. Therefore, the study of the nonstationary random response of random structure is based on a realistic engineering background and is of important theoretical significance, especially in the process of the structural design phase.

Because the random dynamic response analysis of a linear stochastic structure is very complicated and difficult, it is only in recent years that some results have been published. Wall and Bucher[2] studied the dynamic effects of uncertainty in structural properties when the excitation is random by use of the perturbation stochastic finite element method (PSFEM). Liu et al.[3] discussed the secular terms resulting from PSFEM in transient analysis of such a random dynamic system. Jensen and Iwan[4] studied the response of systems with uncertain parameters to random excitation by extending the orthogonal expansion method. Zhao and Chen[5] studied the vibration for structures with stochastic parameters in response to random excitation by using the dynamic Neumann stochastic finite element method, in which the random equation of motion for structure is transformed into a quasi-static equilibrium equation for the solution of displacement in the time domain. Lin et al.[6] studied the stationary random response of a structure with stochastic parameters, in which the random excitations are first transformed into sinusoidal

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*Ph.D., Department of Electronic Mechanical Engineering, P.O. Box 187; gwweigao@yahoo.com.cn.
†Professor, Department of Electronic Mechanical Engineering, P.O. Box 187; jjchen@xidian.edu.cn.
‡Ph.D. Candidate, Department of Electronic Mechanical Engineering.

ones in terms of the pseudoexcitation method (PEM), which turns the joint-random problem into a single random problem for which only structural parameters remain random. Li and Yi[7] expanded the orthogonal expansion method with the PEM for analyzing the dynamic response of structures with uncertain parameters under external random excitation.

The problem of the random dynamic response analysis of linear stochastic structure can be solved by a number of approaches, such as the Monte Carlo simulation method, the stochastic finite element method (SFEM), and the orthogonal expansion method. However, the Monte Carlo simulation method needs a larger amount of computation. The SFEM and the orthogonal expansion method cannot fully reflect the effect of any of the parameters on the structural dynamic response. In this paper, the problems of the nonstationary random dynamic responses of random structures are investigated; a new method the random factor method (RFM), is proposed. Frame structures are taken as objects to be analyzed, in which the randomness of structural physical parameters (elastic modulus and mass density) is considered. The expressions of numerical characteristics of the mean square value of the structural displacement and stress response are developed by means of the random variable's functional moment method and the algebra synthesis method.

The main idea and analyzing procedure of the RFM are as follows. First, a structural parameter random variable is expressed as a random factor multiplied by the determinate value (mean value) of this structural parameter. Second, the structural mass matrix and stiffness matrix are expressed as random factors of structural parameters multiplied by their determinate value (mean value), respectively. Finally, the dynamic characteristics and response are expressed as the functions of these random factors. Therefore, the effect of the randomness of the structural parameters on the structural dynamic response can be reflected expediently and obviously. In addition, we need to analyze the nonstationary response of the traditional determinate structure only one time; then we can obtain the numerical characteristics of the nonstationary response of the random structure by RFM very easily and the computational work is very small.

II. Structural Nonstationary Random Dynamic Response Analysis

Suppose that there are n_e elements in the analyzed frame structure, constructed from one kind of material. Among the physical parameters of structural materials, the influence of the randomness of the Poisson ratio on the structural dynamic responses is insignificant. Therefore, only the randomness of the elastic modulus E and mass density ρ is considered; that is, they are all random variables, and their random variation in space is neglected. In realistic engineering examples, normal distribution is a wide probabilistic distribution, and other probabilistic distributions can be equivalently transformed into normal distribution. Therefore, suppose that these random variables obey normal distribution. The elastic modulus and mass density can be written respectively as $E=\alpha\cdot\tilde{E}$ and $\rho=\beta\cdot\tilde{\rho}$, where α and β are the determinate quantities of E and ρ, respectively; $\tilde{E}$ and $\tilde{\rho}$ are the random variable factors of E and ρ, respectively; the mean values of $\tilde{E}$ and $\tilde{\rho}$ are all 1.0; ν_E and ν_ρ are the variation coefficients of $\tilde{E}$ and $\tilde{\rho}$, respectively.

The eth element's stiffness matrix in local coordinate can be expressed as

$$\left[K^{(e)}\right]=\iiint\limits_V [B]^T[D][B]\,\mathrm{d}V \tag{1}$$

Because the randomness of the structural elastic matrix is determined by the randomness of the elastic modulus, the structural elastic matrix can be expressed as

$$[D]=\tilde{E}[D]^{\#} \tag{2}$$

where $[D]^{\#}$ is the determinate part in the structural elastic matrix, taking the parameters as $E=\alpha$.

Substituting Eq. (2) into Eq. (1), the eth element's stiffness matrix becomes

$$\left[K^{(e)}\right]=\tilde{E}\cdot\iiint\limits_V [B]^T[D]^{\#}[B]\,\mathrm{d}V=\tilde{E}\cdot\left[K^{(e)}\right]^{\#} \tag{3}$$

Furthermore, the structural stiff matrix in global coordinate can be expressed as

$$[K]=\sum_{e=1}^{\eta_e}\left[\hat{K}^{(e)}\right]=\sum_{e=1}^{n_e}\tilde{E}\cdot\left[\hat{K}^{(e)}\right]^{\#}=\tilde{E}[K]^{\#} \tag{4}$$

where $[\hat{K}^{(e)}]^{\#}$ and $[K]^{\#}$ are the determinate parts of $[\hat{K}^{(e)}]$ and $[K]$, respectively.

Likewise, the mass matrix of the structure can be constructed:

$$[M]=\sum_{e=1}^{\eta_e}\left[M^{(e)}\right]=\sum_{e=1}^{\eta_e}\tilde{\rho}\cdot\left[M^{(e)}\right]^{\#}=\tilde{\rho}[M]^{\#} \tag{5}$$

where $[M^{(e)}]^{\#}$ and $[M]^{\#}$ are the determinate parts of $[M^{(e)}]$ and $[M]$, taking the parameters as $\rho=\beta$.

In PSFEM, the stiffness matrix and mass matrix can also be expressed, respectively, as $[M]=[M]^{\#}+\varepsilon_1[M]^{\#}$ and $[K]=[K]^{\#}+\varepsilon_2[K]^{\#}$. However, the parameters ε_1 and ε_2 are small values but not random variables, and so they cannot reflect the randomness of structural parameters directly and obviously. In RFM, the structural parameter variable and its random factor obey the same probabilistic distribution. Thus, there are obvious and huge differences between PFSEM and RFM.

The equation of motion of the structure is given by

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=g(t)\{P(t)\} \tag{6}$$

where $g(t)$ denotes the nonstationary characteristic of the random force. When $g(t)=1(-\infty<t<\infty)$, the applied force is a stationary random force. This is a special nonstationary excitation whose nonstationary property only depends on time and is independent of frequency. The general representation of a nonstationary excitation is $g\,(\omega,t)\,\{P(t)\}$.

Equation (6) is a set of differential equations coupled to each other. Suppose that the structure is initially at rest; then its formal solution can be obtained in terms of the decoupling transform and Duhamel integral. That is,

$$\{u(t)\}=\int_0^t[\phi][h(t-\tau)][\phi]^T g(\tau)\{P(\tau)\}\mathrm{d}\tau \tag{7}$$

where $[\phi]=[\phi_1\ \cdots\ \phi_n]$, $[h(t)]$ can be expressed as

$$[h(t)]=\mathrm{diag}(h_j(t)) \tag{8}$$

$$h_j(t)=\begin{cases}\dfrac{1}{\bar{\omega}_j}\exp(-\xi_j\omega_j t)\sin\bar{\omega}_j t, & t\ge 0\\ 0, & t<0\end{cases}\qquad j=1,2,\ldots,n \tag{9}$$

where $\bar{\omega}_j=\omega_j(1-\xi_j^2)^{\frac{1}{2}}$.

From Eq. (7), the correlation function matrix of the displacement response of the structure can be obtained:

$$\begin{aligned}[R_u(t_1,t_2)]&=E\left(\{u(t_1)\}\{u(t_2)\}^T\right)\\&=\int_0^{t_1}\int_0^{t_2}[\phi][h(t-\tau_1)][\phi]^T g(\tau_1)[R_P(\tau_1-\tau_2)]g(\tau_2)\\&\quad\times[\phi][h(t-\tau_2))]^T[\phi]^T\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2\end{aligned} \tag{10}$$

By performing a Fourier transformation to $[R_u(t_1,t_2)]$, the power spectral density matrix of the displacement response can be obtained as follows:

$$[S_u(\omega)] = [\phi][H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)][\phi]^T \quad (11)$$

$$[H(\omega)] = \mathrm{diag}\lfloor H_j(\omega)\rfloor \quad (12)$$

where

$$H_j(\omega) = 1\Big/\left(\omega_j^2 - \omega^2 + i\cdot 2\xi_j\omega_j\omega\right)$$
$$i = \sqrt{-1}, \qquad j = 1, 2, \ldots, n \quad (13)$$

After integrating $[S_u(\omega)]$ within the frequency domain, the mean square value matrix of the structural displacement response can be obtained:

$$\left[\psi_u^2\right] = \int_0^\infty [S_u(\omega)]\,\mathrm{d}\omega = \int_0^\infty [\phi][H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2) \times [\phi][H^*(\omega)][\phi]^T\,\mathrm{d}\omega \quad (14)$$

Then the mean square value of the kth degree of freedom of the structural dynamic displacement response can be expressed as

$$\psi_{uk}^2 = \phi_k\cdot\int_0^\infty [H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)]\,\mathrm{d}\omega\cdot\phi_k^T \qquad k = 1, 2, \ldots, n \quad (15)$$

where ϕ_k is the kth line vector of the matrix $[\phi]$.

According to the relationship between node displacement and element stress, the stress response of the eth element in the frame structure is

$$\left\{\sigma(t)^{(e)}\right\} = \tilde{E}\cdot[D]^\#\cdot[B]\cdot\left\{u(t)^{(e)}\right\}, \qquad e = 1, 2, \ldots, n_e \quad (16)$$

From Eq. (16), the correlation function matrix of the eth element stress response can be obtained:

$$\left[R_\sigma^{(e)}(t_1, t_2)\right] = E\left(\left\{\sigma(t_1)^{(e)}\right\}\left\{\sigma(t_2)^{(e)}\right\}^T\right) = \tilde{E}[D]^\#[B]\left[R_u^{(e)}(t_1, t_2)\right][B]^T[D]^{\#^T}\tilde{E} \quad (17)$$

Furthermore, the power spectral density matrix of the stress response of the eth element can be obtained:

$$\left[S_\sigma^{(e)}(\omega)\right] = \tilde{E}[D]^\#[B]\left[S_u^{(e)}(\omega)\right][B]^T[D]^{\#^T}\tilde{E} \quad (18)$$

Then the mean square value matrix of the eth element stress response can be expressed as

$$\left[\psi_{\sigma^{(e)}}^2\right] = \tilde{E}[D]^\#[B]\left[\psi_{u^{(e)}}^2\right][B]^T[D]^{\#^T}\tilde{E} \quad (19)$$

III. Numerical Characteristics of the Stationary Random Response of the Random Structure

A. Numerical Characteristics of Natural-Frequency Random Variable

From Eqs. (4) and (5), it can be obtained that $[M]$ and $[K]$ are random variables. The randomness of the physical parameters will lead to the randomness of the natural frequency ω. In the following, the structural dynamic characteristic based on probability will be analyzed.

Suppose that the jth-order natural frequency and mode shape of the structure are ω_j and $\{\phi\}_j$, respectively. By using the RFM, ω_j and $\{\phi\}_j$ can be written, respectively, as

$$\omega_j = \tilde{\omega}_j\cdot\omega_j^\# \quad (20)$$

$$\{\phi\}_j = \tilde{\phi}_j\cdot\{\phi\}_j^\# \quad (21)$$

By using Rayleigh's quotient expression, the random variable of jth natural frequency can be expressed as

$$\omega_j^2 = \frac{\{\phi\}_j^T[K]\{\phi\}_j}{\{\phi\}_j^T[M]\{\phi\}_j} \quad (22)$$

Substituting Eqs. (4), (5), and (21) into Eq. (20) yields

$$\omega_j^2 = \frac{\tilde{\phi}_j\cdot\tilde{E}\cdot\tilde{\phi}_j}{\tilde{\phi}_j\cdot\tilde{\rho}\cdot\tilde{\phi}_j}\cdot\frac{\{\phi\}_j^{\#^T}[K]^\#\{\phi\}_j^\#}{\{\phi\}_j^{\#^T}[M]^\#\{\phi\}_j^\#} = \frac{\tilde{E}}{\tilde{\rho}}\cdot\frac{K_j^\#}{M_j^\#} = \frac{\tilde{E}}{\tilde{\rho}}\left(\omega_j^\#\right)^2 \quad (23)$$

where $K_j^\#$, $M_j^\#$, and $\omega_j^\#$ are all determinate quantities: they are the jth-order modal stiffness, modal mass, and natural frequency of structure when $E = \alpha$ and $\rho = \beta$, respectively.

Comparing Eq. (20) with Eq. (23), it can be easily obtained that

$$\tilde{\omega}_j = (\tilde{E}/\tilde{\rho})^{\frac{1}{2}} \quad (24)$$

By means of the algebra synthesis method,[8] the computational expressions of mean value and mean variance of jth order natural frequency can be deduced as follows:

$$\mu_{\omega_j} = \omega_j^\# \sqrt{\tfrac{1}{2}(\mu_{\tilde{E}}/\mu_{\tilde{\rho}})\sqrt{4\left[1+\nu_\rho(\nu_\rho - C_{E\rho}\nu_E)\right]^2 - 2\left(\nu_E^2+\nu_\upsilon^2-2C_{E\rho}\nu_E\nu_\rho\right)}} \quad (25)$$

$$\sigma_{\omega_j} = \omega_j^\#\Big\{(\mu_{\tilde{E}}/\mu_{\tilde{\rho}})[1+\nu_\rho(\nu_\rho - C_{E\rho}\nu_E)] - \tfrac{1}{2}(\mu_{\tilde{E}}/\mu_{\tilde{\rho}}) \times\sqrt{4[1+\nu_\rho(\nu_\rho - C_{E\rho}\nu_E)]^2 - 2\left(\nu_E^2+\nu_\upsilon^2-2C_{E\rho}\nu_E\nu_\rho\right)}\Big\}^{\frac{1}{2}} \quad (26)$$

If variables E and ρ are independent, then $C_{E\rho} = 0$. If variable E is completely correlative with ρ, then $C_{E\rho} = 1$. These are two kinds of extreme situations. According to observation of the property of common materials, it can be found that the elastic modulus E is usually positively correlated with mass density ρ and that degree of correlation is rather high. Therefore, in practical computation, it is suggested that the correlative coefficient $C_{E\rho} = 0.5\sim 0.9$ (Refs. 8 and 9).

From the preceding formulas, it is easily seen that the values of the variation coefficient (randomness) of every natural frequency ν_{ω_j} are equal to each other. They only depend on the physical parameters E and ρ, as well as the correlative coefficient $c_{E\rho}$, but they are independent of the order number of the structural vibration model. The bigger the mean values of the structural higher order natural frequencies are, the bigger their mean variances are. Furthermore, a change in the properties of a structure will have a larger effect for higher natural frequencies; that is, their dispersal degree is bigger. According to the determinate stiffness and mass matrices, $[K]^\#$ and $[M]^\#$, the determinate values of every order of natural frequency $\omega_j^\#$ can be obtained by means of the conventional dynamic analysis method.

The randomness of the structural dynamic characteristics and the stochastic excitation will lead to the randomness of the mean square value of structural dynamic response (dynamic displacement and dynamic stress). The statistical description of random variables is represented by using its numerical characteristic. In the following, the expressions of numerical characteristics of the structural nonstationary-response random variables will be derived.

B. Numerical Characteristics of the Nonstationary Random Response of the Random Structure

By means of the random variable's functional moment method,[10] the mean value and mean variance of the mean square value of the kth degree of freedom of the structural dynamic displacement response can be deduced from Eq. (15):

$$\mu_{\psi^2_{uk}} = \mu_{\bar{\phi}_k} \cdot \int_0^\infty \mu_{[H(\omega)]}\mu_{[\phi]^T} g(t_1)\mu_{[S_P(\omega)]} g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]} \times \mathrm{d}\omega \cdot \mu_{\bar{\phi}_k^T} \tag{27}$$

$$\sigma_{\psi^2_{uk}} = \mu_{\bar{\phi}_k} \cdot \left\{ \int_0^\infty \left\{ \mu^2_{[H(\omega)]} \left(\mu_{[\phi]^T} g(t_1)\mu_{[S_P(\omega)]} g(t_2)\mu_{[\phi]}\right)^2 \sigma^2_{[H^*(\omega)]} + \sigma^2_{[H(\omega)]} \left(\mu_{[\phi]^T} g(t_1)\mu_{[S_P(\omega)]} g(t_2)\mu_{[\phi]}\right)^2 \mu^2_{[H^*(\omega)]} + \sigma_{[H(\omega)]}\mu_{[\phi]^T} g(t_1)\mu_{[S_P(\omega)]} g(t_2)\mu_{[\phi]}\sigma_{[H^*(\omega)]} \right\} \mathrm{d}\omega \right\}^{\frac{1}{2}} \cdot \mu_{\bar{\phi}_k^T}$$

$$k = 1, 2, \ldots, n \tag{28}$$

$$\sigma_{[H(\omega)]} = \mathrm{diag}\left[\frac{-2\mu_{\omega_j} - i \cdot 2\xi_j \omega}{\left(\mu^2_{\omega_j} - \omega^2 + i \cdot 2\xi_j \mu_{\omega_j}\omega\right)^2} \cdot \sigma_{\omega_j} \right]$$

$$j = 1, 2, \ldots, n \tag{29}$$

From Eqs. (27) and (28), the variation coefficient $\nu_{\psi^2_{uk}}$ of the random variable ψ^2_{uk} can be obtained:

$$\nu_{\psi^2_{uk}} = \sigma_{\psi^2_{uk}} \big/ \mu_{\psi^2_{uk}} \tag{30}$$

From Eq. (19), the mean value and mean variance of the mean square value of the eth element stress response can be deduced by means of the algebra synthesis method:

$$\mu_{\left[\psi^2_{\sigma(e)}\right]} = \left(\mu^2_{\tilde{E}} + \sigma^2_{\tilde{E}}\right) \cdot [D]^\#[B] \cdot \mu_{\left[\psi^2_{u(e)}\right]} \cdot [B]^T [D]^{\#^T} \tag{31}$$

$$\sigma_{\left[\psi^2_{\sigma(e)}\right]} = \left\{ \left(\mu^2_{\tilde{E}} + \sigma^2_{\tilde{E}}\right)^2 \cdot \left([D]^\#[B] \cdot \sigma_{\left[\psi^2_{u(e)}\right]} \cdot [B]^T [D]^{\#^T}\right)^2 + \left(4\mu^2_{\tilde{E}}\sigma^2_{\tilde{E}} + 2\sigma^4_{\tilde{E}}\right) \cdot \left([D]^\#[B] \cdot \mu_{\left[\psi^2_{u(e)}\right]} \cdot [B]^T [D]^{\#^T}\right)^2 + \left(4\mu^2_{\tilde{E}}\sigma^2_{\tilde{E}} + 2\sigma^4_{\tilde{E}}\right) \cdot \left([D]^\#[B] \cdot \sigma_{\left[\psi^2_{u(e)}\right]} \cdot [B]^T [D]^{\#^T}\right)^2 \right\}^{\frac{1}{2}} \tag{32}$$

From Eqs. (31) and (32), the variation coefficient of the mean square value of the eth element stress response can be obtained:

$$\nu_{\left[\psi^2_{\sigma(e)}\right]} = \sigma_{\left[\psi^2_{\sigma(e)}\right]} \Big/ \mu_{\left[\psi^2_{\sigma(e)}\right]} \tag{33}$$

Here we should indicate that this analysis method is only applicable for the random structure in which the randomness of Youngs modulus and mass densities of each frame are equal to each other; that is, the random variables are the same in all structural members.

IV. Example

According to the preceding computational formulas and the solution method, the corresponding computational program is designed. A 12-bar planar frame structure is utilized as an example (Fig. 1). The material of this structure is steel. Its elastic modulus E and mass density ρ are all random variables: $\mu_E = 2.058 \times 10^5$ MPa, $\mu_\rho = 76.5$ KN/m^3, and let $\xi_j = \xi = 0.01$. Ground-level accelerations of an earthquake act on the frame structure. Here, the model of Kanai-Tajimi is selected. $P(t)$ is a Gauss stochastic process and its mean value is zero, and its one-sided self-power spectral density can be expressed as[7]

$$S_{PP}(\omega) = \frac{1 + 4(\xi_g\omega/\omega_g)^2}{\left(1 - \omega^2/\omega_g^2\right)^2 + 4(\xi_g\omega/\omega_g)^2} S_0 \tag{34}$$

Table 1 Numerical characteristics of the displacement response

Model	$\mu_{\psi^2_{X9}}$, mm^2	$\sigma_{\psi^2_{X9}}$, mm^2	$\nu_{\psi^2_{X9}}$
$\nu_E = 0.1, \nu_\rho = 0$	2.9862	0.1372	0.04594
	2.9871[a]	0.1375[a]	0.04603[a]
$\nu_\rho = 0.1, \nu_E = 0$	2.9862	0.1369	0.04584
	2.9871[a]	0.1372[a]	0.04592[a]
$\nu_E = \nu_\rho = 0.1$	2.9862	0.1694	0.05673
	2.9877[a]	0.1698[a]	0.05682[a]
$\nu_E = 0.2, \nu_\rho = 0$	2.9862	0.2715	0.09092
	2.9882[a]	0.2722[a]	0.09111[a]
$\nu_\rho = 0.2, \nu_E = 0$	2.9862	0.2709	0.09072
	2.9882[a]	0.2716[a]	0.09091[a]
$\nu_E = \nu_\rho = 0.2$	2.9862	0.3408	0.1141
	2.9887[a]	0.3416[a]	0.1143[a]

[a]Monte Carlo simulation method.

Table 2 Numerical characteristics of the stress response

Model	$\mu_{\psi^2_{\sigma1}}$, Mp^2	$\sigma_{\psi^2_{\sigma1}}$, Mp^2	$\nu_{\psi^2_{\sigma1}}$
$\nu_E = 0.1, \nu_\rho = 0$	1789.4	371.8	0.20778
$\nu_\rho = 0.1, \nu_E = 0$	1741.5	29.04	0.01667
$\nu_E = \nu_\rho = 0.1$	1836.7	401.9	0.21882
$\nu_E = \nu_\rho = 0.2$	2154.9	853.9	0.39626
	2169.2[a]	859.7[a]	0.39631[a]

[a]Monte Carlo simulation method.

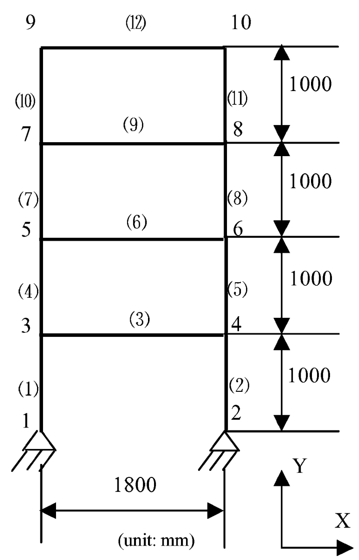

Fig. 1 Twelve-bar planar frame structure.

where $\omega_g = 16.5$, $\xi_g = 0.7$, and $S_0 = 15.6$ cm^2/s^3; $g(t)$ is the time modulation function and can be expressed as[7]

$$g(t) = \begin{cases} (t/t_b)^2, & 0 \le t < t_b \\ 1.0, & t_b \le t < t_c \\ \exp[-\lambda(t - t_c)], & t \ge t_c \end{cases} \tag{35}$$

where $t_b = 7.1$ s, $t_c = 19.5$ s, and $\lambda = 0.16$.

To investigate the effect of the dispersal degree of random variables E and ρ on the structural dynamic response, the different models are selected and the values of variation coefficients of parameters E and ρ are taken as different groups. For $t = 15$ s the computational results of the mean value $\mu_{\psi^2_{X9}}$, mean variance $\sigma_{\psi^2_{X9}}$, and variation coefficient $\nu_{\psi^2_{X9}}$ of the mean square value of displacement response ψ^2_{X9} of node 9 of the X direction are listed in Table 1. In addition, to verify the RFM, the computational results that are obtained by the Monte Carlo simulation method are given in Table 1, in which 3000 simulations are used.

When $t = 15$ s, the computational results of the mean value $\mu_{\psi^2_{\sigma1}}$, mean variance $\sigma_{\psi^2_{\sigma1}}$, and variation coefficient $\nu_{\psi^2_{\sigma1}}$ of the mean square value of the stress response $\psi^2_{\sigma1}$ of the first element obtained by RFM and Monte Carlo simulation method are listed in Table 2.

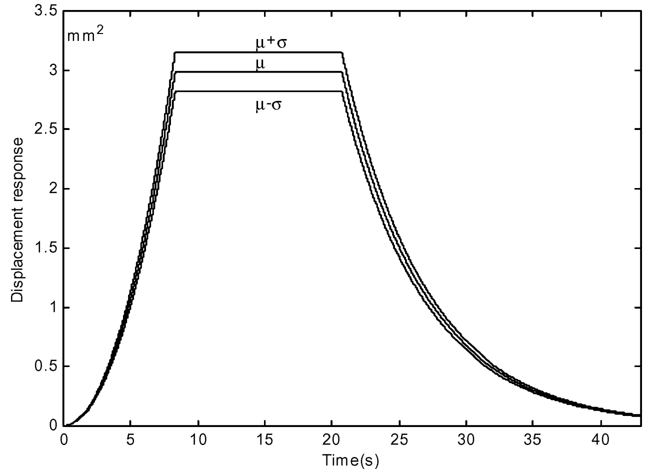

Fig. 2 MSVSDR of N9-XD ($\nu_E = \nu_\rho = 0.1$).

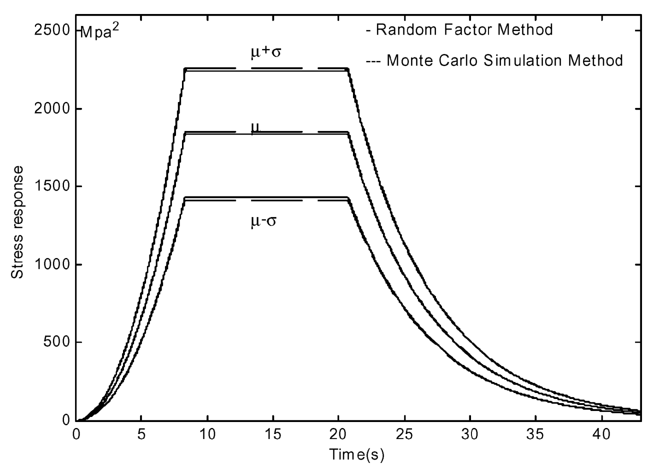

Fig. 3 MSVSSR of E1 ($\nu_E = \nu_\rho = 0.1$).

The course curves of the mean value (μ), mean value plus mean variance ($\mu + \sigma$), and mean value − mean variance ($\mu - \sigma$) of the mean square value of the structural displacement response (MSVSDR) of node 9 of the X direction (N9-XD) are shown in Fig. 2. The course curves of the mean value (μ), mean value plus mean variance ($\mu + \sigma$), and mean value minus mean variance ($\mu - \sigma$) of the mean square value of structural stress response (MSVSSR), of the first element (E1) obtained by our method and the Monte Carlo simulation method are shown in Fig. 3.

From Table 1 to Table 2 and Fig. 2 to Fig. 3, the following can be seen easily:

1) The analytic results of the method proposed in this paper agree with that of the Monte Carlo simulation method, by which the validity of our method is verified. Because the high-order small values of the mean variance are neglected by using the random variable's functional moment method, the mean variance and variation coefficient obtained via the Monte Carlo method are bigger.

2) The effects of the randomness of E and ρ on the randomness of the mean square value of the structural displacement and stress response are different. The effects of the randomness of E and ρ on the randomness of the MSVSDR are almost same; however, the randomness of the elastic modulus E affects the randomness of the MSVSSR more than the randomness of ρ does.

3) Compared with the case that only one value of the randomness of E or ρ is taken into account, the randomness of the structural dynamic characteristics is greater if their randomnesses are considered simultaneously.

4) The dispersal degree of the structure's dynamic response will increase along with the increase of the variation coefficients of the structural physical parameters.

V. Conclusions

The computational expressions of the mean value, mean variance, and variation coefficient of the mean square value of the structural displacement and stress response under the nonstationary random excitation are developed in this paper; the dynamic response analysis results of the random structure to the stochastic excitation can be obtained expediently. The examples show that the model and method of the nonstationary random dynamic response analysis of random structure presented in this paper is rational and feasible. In addition, this method can also be applied to the nonstationary random dynamic response analysis of other types of random structures.

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A. Berman
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